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Squeezed states and the quantum noise of light in semiconductor microcavities

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Abstract. A theoretical investigation of the quantum noise in the light reflected by a microcavity containing a semiconductor quantum well is presented. Squeezing is predicted when scattering processes have a low efficiency. Exciton–phonon scattering is shown to destroy the non-classical effects and to yield excess noise in the output field.

1. Introduction

In recent years, the optical properties of microcavities containing semiconductor quantum wells have been the subject of detailed investigations. Up to now interest has been focused mostly on the spectral properties of the reflected, transmitted and emitted light, and on non-linear optical properties [1]. Quantum properties of the reflected or emitted light have been much less studied. Quantum effects such as squeezing and antibunching of the outgoing light have been predicted and observed in microcavities containing atoms [2]. Because of the similarities between atomic and excitonic resonances, it can be argued that a semiconductor microcavity should also give rise to such effects. Two main features are necessary: first, modifying the quantum statistical properties of light requires a coherent non-linearity in the system; second, the non-classical features must not be destroyed by spurious fluctuations linked to the relaxation processes present in semiconductors. In spite of numerous non-radiative relaxation processes that cause a fast decay of coherences, semiconductors have already been shown to exhibit coherent non-linear effects [3], such as the dynamical Stark shift [4]. Furthermore, recent experiments have demonstrated the possibility of modifying the quantum fluctuations and of generating squeezing in semiconductors [5].

In this paper, we present a theoretical study of the quantum fluctuations of the light going out of a microcavity containing one semiconductor quantum well. Our model assumes a non-linearity arising from the exciton–exciton interaction [6]. In contrast to previous treatments [7], we consider incoming fluctuations that are not only zero-point fluctuations, but also fluctuations related to the existing relaxation processes, as required by the fluctuation-dissipation theorem. Calculations are performed using realistic parameters found in the present-day state-of-the-art semiconductor microcavities. We show that at low temperature when interaction with phonons is small, reduction of the quantum fluctuations of the reflected light is expected.

Engineering of the quantum fluctuations of light in semiconductor materials would open the way to compact quantum devices, such as noiseless sources of light, thresholdless lasers or highly efficient all-optical switches. The understanding of quantum properties of semiconductor microcavities is thus of great importance.

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2. The model

We consider a microcavity containing a semiconductor quantum well embedded between two highly reflecting planar mirrors separated by a distance of the order of the wavelength. The discussion is limited to a two-band semiconductor. The electromagnetic field can excite an electron from the filled valence band to the conduction band, thereby creating a hole in the valence band. The electron–hole system possesses bound states, the excitonic states. We will only consider the lowest of these bound states, the 1s state.

Neglecting the spin degrees of freedom, we can write an effective interaction Hamiltonian for the coupled exciton–photon system in the cavity as [6, 8, 9]

$$\begin{aligned}H = {}& \sum_k \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k + \sum_K \hbar\omega_K \hat{b}_K^\dagger \hat{b}_K + \sum_k \hbar g_k (\hat{a}_k^\dagger \hat{b}_k + \hat{b}_k^\dagger \hat{a}_k) \\ & + \sum_{K,K'} \sum_Q \hbar\alpha_{KK'Q} \hat{b}_K^\dagger \hat{b}_{K'}^\dagger \hat{b}_{K+Q} \hat{b}_{K'-Q} \\ & + \left(\sum_{K,K'} \sum_Q \hbar\alpha'_{KK'Q} \hat{b}_K^\dagger \hat{b}_{K'}^\dagger \hat{b}_{K+Q} \hat{a}_{K'-Q} + \text{h.c.} \right) \\ & + \sum_{KK'} \beta_{KK'} \hat{b}_K^\dagger \hat{b}_{K'} (\hat{c}_{K-K'} + \hat{c}_{K'-K}^\dagger) + \left(\sum_{kj} \gamma_{kj} \hat{a}_k^\dagger \hat{\tau}_j + \text{h.c.} \right). \end{aligned} \quad (1)$$

As the exciton and photon modes are quantized along the direction normal to the microcavity, we consider the lowest-order mode in this direction, and the sums over k and K run for the momenta in the cavity plane only. The first two terms correspond to the energies of the photons and of the excitons, where $\hat{a}_k$ and $\hat{b}_K$ are respectively the annihilation operators of a photon of in-plane momentum k and of an exciton of in-plane momentum K, and ω_k and ω_K are the frequencies of the corresponding cavity and exciton modes. The third term corresponds to the exciton–photon coupling with a strength g_k. Due to the translational invariance in the plane of the semiconductor layers, excitons with a wave vector K in this plane can only be coupled with light having an equal in-plane wave vector $k = K$.

The term in the second line describes the exciton–exciton scattering due to Coulomb interaction, while the term in the third line represents the saturation of the photon–exciton interaction. The first term in the fourth line describes the exciton–phonon scattering, where $\hat{c}_Q$ is the phonon annihilation operator. The second term represents the coupling between the electromagnetic field modes inside the cavity, $\hat{a}_k$, and outside the cavity, $\hat{\tau}_j$, the latter being considered as a reservoir.

We deal with the case of only one photon mode irradiating the microcavity, for which we will assume $k = 0$. Because of the in-plane momentum conservation in the exciton–photon interaction, this cavity mode is only coupled with one exciton mode of $K = 0$. We will be interested in the case of strong coupling between excitons and photons in this mode, which leads to remarkable properties of microcavities [10]. All of the other exciton modes form a reservoir that is weakly coupled to the mode of interest.

The interaction between the exciton and the photon modes of interest can then be modelled by the coupling of two harmonic oscillators, together with an excitonic non-linearity coming from the terms in the second line of equation (1) with $K = K' = Q = 0$. The Hamiltonian of the coupled system can be written as

$$H = \hbar\omega_{cav} \hat{a}^\dagger \hat{a} + \hbar\omega_{exc} \hat{b}^\dagger \hat{b} + \hbar g (\hat{a}^\dagger \hat{b} + \hat{b}^\dagger \hat{a}) + \hbar\alpha \hat{b}^\dagger \hat{b}^\dagger \hat{b} \hat{b} + H_{rel}. \quad (2)$$

The non-linear term describing saturation effects will not be treated here. It can be shown that it gives rise to small corrections as compared to the previous one [11]. The term H_{rel}

contains all of the other terms of equation (1) and gives rise to relaxation of the main exciton and photon modes. The quantum theory of damping can then be used to derive the dissipation terms and fluctuation terms associated with H_{rel} [12, 13].

The problem of the determination of the quantum optical properties of the outgoing field has some similarities with the one of a cavity containing atoms. The squeezing properties of the field going out of a cavity containing atoms in the strong-coupling regime were investigated in reference [14]. Here we will compute the squeezing spectra of the output field of a microcavity containing excitons, for which the non-linearity is different from the atomic one. This case was not investigated in detail previously. Moreover, we will study the effect of the presence of a thermal reservoir of phonons coupled to the system.

The microcavity is irradiated by a coherent field from a laser of frequency ω_L. One mirror of the microcavity is perfectly reflecting, whereas the other one, having a small non-zero transmission coefficient, is the coupling mirror, through which the light is coupled in and out. Including relaxation processes, one can derive from the Hamiltonian (2) evolution equations for the time-dependent electromagnetic field and exciton field operators inside the cavity:

$$\frac{\mathrm{d}\hat{a}}{\mathrm{d}t} = -(\gamma_a + \mathrm{i}\delta_a)\hat{a} - \mathrm{i}g\hat{b} + \sqrt{2\gamma_a}a^{\mathrm{in}} \tag{3}$$

$$\frac{\mathrm{d}\hat{b}}{\mathrm{d}t} = -(\gamma_b + \mathrm{i}\delta_b)\hat{b} - \mathrm{i}g\hat{a} - 2\mathrm{i}\alpha\hat{b}^{\dagger}\hat{b}\hat{b} + \sqrt{2\gamma_b}b^{\mathrm{in}} \tag{4}$$

where t is a dimensionless time normalized to the round-trip time τ_c in the cavity, γ_a and γ_b are the dimensionless decay constants of the cavity field and of the exciton, i.e. the cavity field and exciton decay rates normalized to $1/\tau_c$, $\delta_a = (\omega_{cav} - \omega_L)\tau_c$ and $\delta_b = (\omega_{exc} - \omega_L)\tau_c$ are the dimensionless detunings of the cavity and of the exciton from the frequency ω_L of the incoming laser field. The exciton-to-photon coupling constant g and the non-linear coupling constant α have also been normalized to $1/\tau_c$. The fields $\hat{a}^{\mathrm{in}}$ and $\hat{b}^{\mathrm{in}}$ are the incoming electromagnetic and exciton fields, the characteristics of which will be discussed below.

The relaxation of the field in the cavity is related to the last term in equation (1). The decay constant γ_a is linked to the amplitude reflection coefficient r of the coupling mirror by $r = 1-\gamma_a$. Since the cavity has a high finesse, r is close to 1, which implies that the amplitude transmission coefficient t is much smaller than 1 and verifies $t = \sqrt{2\gamma_a}$.

We will solve the problem in the framework of the input–output formalism [15] where the evolution of the fields for the electromagnetic and exciton modes is computed using equations (3), (4) while the output field is related to the intracavity and input fields by

$$\hat{a}^{\mathrm{out}} = \sqrt{2\gamma_a}\hat{a} - \hat{a}^{\mathrm{in}}. \tag{5}$$

Equation (5) indicates that the outgoing field is the sum of the inside field transmitted through the coupling mirror and of the input field reflected by the mirror (the reflection coefficient r has been replaced by 1 in this equation).

The relaxation of the exciton is due to exciton–exciton scattering represented by the terms of the second line of equation (1) and to scattering of excitons by acoustic phonons represented by the first term in the fourth line of equation (1). The first process is a density-dependent process, while the second one is mainly sensitive to temperature. At low enough temperature and with good-quality samples, we assume that we can neglect all other relaxation processes. The contribution of these two processes to the decay constant γ_b can be calculated from the interaction Hamiltonian [16, 17]. Let us note that the electron–hole radiative recombination, occurring only with photons having $k = K$, is accounted for by the cavity–exciton coupling term.

Equations (3), (4) can be considered as Langevin equations for the two fields, where the fluctuating parts of the terms $\sqrt{2\gamma_a}a^{\mathrm{in}}$ and $\sqrt{2\gamma_b}b^{\mathrm{in}}$ are the Langevin forces associated with the reservoirs for the electromagnetic field and for the excitons. The minimal values of these fluctuating terms correspond to the zero-point fluctuations (or vacuum fields) imposed by quantum mechanics. Additional fluctuations may arise from the specific scattering processes. While non-classical optical effects have been predicted by several authors in the case of vacuum incoming fluctuations [7], we evaluate the squeezing and modifications of quantum fluctuations when incoming fluctuations related to actual dissipation processes are included. As a model case, we will concentrate on the fluctuations coming from phonon scattering.

The incoming electromagnetic field is the laser coherent field, which is a classical field together with quantum fluctuations equal to the vacuum fluctuations. Thus we have $\hat{a}^{\mathrm{in}} = a^{\mathrm{in}} + \delta\hat{a}^{\mathrm{in}}$ where a^{in} is the classical mean value of the field and its fluctuations $\delta\hat{a}^{\mathrm{in}}$ have a zero mean value. The only non-zero correlation function of the fluctuations is

$$\langle \delta\hat{a}^{\mathrm{in}}(t)\, \delta\hat{a}^{\mathrm{in}\dagger}(t')\rangle = \delta(t - t'). \tag{6}$$

The exciton field inside the cavity is coupled with a fluctuating field $\hat{b}^{\mathrm{in}} = \delta\hat{b}^{\mathrm{in}}$ that has several contributions associated with the various relaxation processes. Here, we will only consider the term coming from phonon scattering, which can be treated as the coupling with a thermal bath, as shown below. The fluctuating terms associated with exciton–exciton scattering and their correlation functions have been computed in reference [13]. Their contribution to the output noise can be calculated from [18]. It is usually smaller than the one coming from exciton–phonon scattering and will be treated elsewhere.

As can be seen from the first term in the third line of equation (1) describing exciton–phonon scattering, an exciton in the mode of interest b is annihilated while an exciton in another mode is created and a phonon is created or annihilated, the energy and momentum conservation being ensured by the phonon. This term can be treated by considering that the excitons interact with a thermal bath [13, 19], the temperature of which depends on the experimental conditions. The only two non-zero correlation functions are then given by

$$\langle \delta\hat{b}^{\mathrm{in}}(t)\, \delta\hat{b}^{\mathrm{in}\dagger}(t')\rangle = (1 + \langle n\rangle)\delta(t - t') \tag{7}$$

$$\langle \delta\hat{b}^{in\dagger}(t)\, \delta\hat{b}^{\mathrm{in}}(t')\rangle = \langle n\rangle\delta(t - t'). \tag{8}$$

$\langle n\rangle$ is the mean number of excitations in the thermal bath. We assume that the phonon-mediated coupling of the exciton b-mode occurs mainly with the ensemble of non-radiative excitons whose energies differ from the energy of the mode studied by an energy of the order of the vacuum Rabi splitting ΔE [20, 21]. The phonons capable of matching the energy difference ΔE have a mean occupation number

$$\langle n\rangle = \frac{1}{\mathrm{e}^{\Delta E/kT} - 1}. \tag{9}$$

At zero temperature $T = 0$ we have $\langle n\rangle = 0$. The only non-zero correlation function is given by equation (7) and corresponds to zero-point fluctuations of the system.

To calculate the fluctuations of the outgoing light field, we will linearize equations (2), (3) in the vicinity of the operating point. To do so, we first compute the mean values of the fields.

3. Mean fields

In order to study the mean values of the electromagnetic and excitonic fields, we rewrite equations (3), (4) for classical values of the field, removing the fluctuating terms, and we solve them in the steady-state regime ($\mathrm{d}a/\mathrm{d}t = \mathrm{d}b/\mathrm{d}t = 0$). The equations are

$$(\gamma_a + \mathrm{i}\delta_a)a + \mathrm{i}gb = \sqrt{2\gamma_a}a^{\mathrm{in}} \tag{10}$$

$$(\gamma_b + \mathrm{i}\delta_b)b + \mathrm{i}ga = 2\mathrm{i}\alpha b^* b^2. \tag{11}$$

In the linear case ($\alpha = 0$), they yield simple analytical expressions for the mean intensities I_a and I_b of the fields:

$$\frac{I_a}{I^{\rm in}} = \frac{2\gamma_a(\gamma_b^2 + \delta_b^2)}{(g^2 + \gamma_a\gamma_b - \delta_a\delta_b)^2 + (\gamma_a\delta_b + \gamma_b\delta_a)^2} \tag{12}$$

$$\frac{I_b}{I^{\rm in}} = \frac{2\gamma_a g^2}{(g^2 + \gamma_a\gamma_b - \delta_a\delta_b)^2 + (\gamma_a\delta_b + \gamma_b\delta_a)^2} \tag{13}$$

where $I^{\rm in} = |a^{\rm in}|^2$ is the incoming laser field intensity. The intensity $I^{\rm out}$ of the reflected field

$$a^{\rm out} = \sqrt{2\gamma_a}a - a^{\rm in} \tag{14}$$

is given by

$$\frac{I^{\rm out}}{I^{\rm in}} = \frac{(g^2 - \gamma_a\gamma_b - \delta_a\delta_b)^2 + (\gamma_a\delta_b - \gamma_b\delta_a)^2}{(g^2 + \gamma_a\gamma_b - \delta_a\delta_b)^2 + (\gamma_a\delta_b + \gamma_b\delta_a)^2}. \tag{15}$$

This enables us to calculate the reflectivity R and absorption $A = 1 - R$ of the microcavity. The absorption is found to be proportional to the excitonic field intensity:

$$\frac{I_b}{I^{\rm in}} = \frac{A}{2\gamma_b}. \tag{16}$$

Thus the absorption spectrum of the microcavity gives direct access to the variation of I_b with the laser frequency. When the exciton and cavity are in resonance ($\delta_a = \delta_b$), the degeneracy is lifted due to the strong coupling, and the field intensities have two symmetrical peaks. The energy difference between the two peaks yields the vacuum Rabi splitting for the intracavity field, Δ_a, and for the absorption, Δ_b:

$$\Delta_a = 2\sqrt{g\sqrt{g^2 + 2\gamma_b(\gamma_a + \gamma_b)} - \gamma_b^2} \tag{17}$$

$$\Delta_b = 2\sqrt{g^2 - \frac{\gamma_a^2 + \gamma_b^2}{2}}. \tag{18}$$

When non-linear processes are taken into account, there is no simple analytical expression for the field intensities. However, it is possible to write the incoming field intensity as a function of the excitonic field intensity:

$$I^{\rm in} = \frac{1}{2\gamma_a}\frac{(g^2 - \delta_a\delta_b + \gamma_a\gamma_b - \alpha\gamma_a I_b)^2 + (\gamma_a\delta_b + \gamma_b\delta_a - \delta_a\alpha I_b)^2}{g^2} I_b. \tag{19}$$

This expression would lead to bistability for high enough values of the non-linearity. However, this has never been observed experimentally, and the corresponding situation is unrealistic since it occurs with excitations for which higher-order effects in the exciton–exciton interaction take place. Actually experiments showed that in such cases, the bleaching of the oscillator strength causes the strong-coupling effect to disappear [22, 23].

The intracavity electromagnetic field intensities will be expressed in units of a 'saturating' intensity I_0. I_0 is defined as the intracavity electromagnetic intensity yielding a density of excitons of 10^9 excitons cm^{-2} over the active area of 0.1 mm^2 in the absence of non-linear effects. The latter value is usually considered as the limit of the low-density case [22], where our treatment is expected to be valid. The non-linear coefficient α was evaluated from reference [6] to be 1.5×10^{-9} for an active area of 0.1 mm^2.

Figure 1 shows the variation of the intracavity electromagnetic field intensity (a) and the excitonic field intensity (b) with laser detuning when the exciton and cavity are in resonance: $\delta_a = \delta_b$. We have taken equal cavity and exciton widths, $\gamma_a = \gamma_b = 0.05g$. The photon–exciton coupling coefficient g is equal to 2×10^{-2} in units of the inverse round-trip time in the

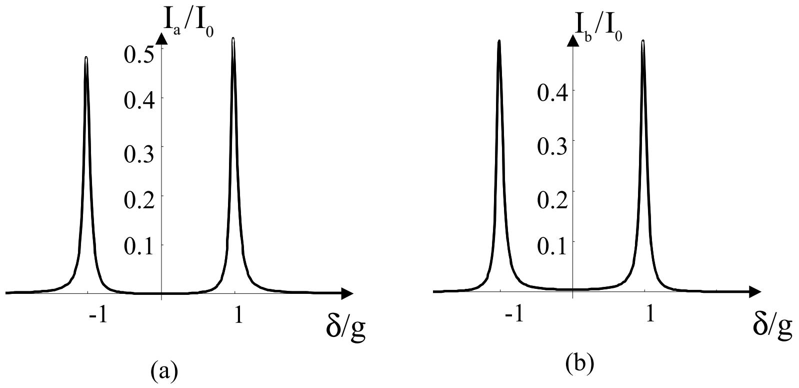

Figure 1. Intensities I_a of the cavity mode (a) and I_b of the exciton mode (b) as functions of the laser detuning δ (normalized to g), when the cavity is resonant with the exciton. The parameters are as follows: $\gamma_a = \gamma_b = 0.05g$, $2\alpha = 1.5 \times 10^{-9}$, $I_m = 0.5I_0$.

microcavity, $2\alpha = 1.5 \times 10^{-9}$, and the intracavity intensity in the absence of non-linearity is $I_m = 0.5I_0$. The heights of the two maxima are slightly different, but the shape of the curve is quite compatible with experimental observations. For the calculation of the squeezing effects, we will focus on such cases of weak non-linear effects, in which the peaks are not changed from their shapes in the absence of non-linearity.

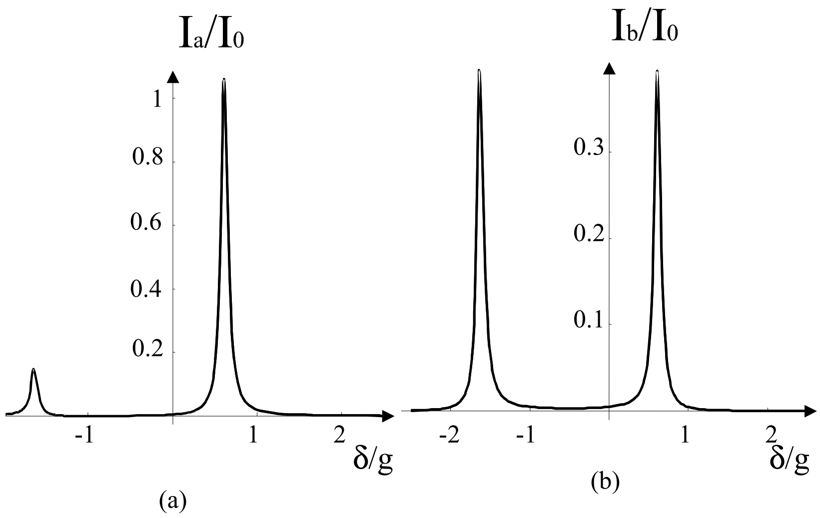

Figure 2. Intensities I_a of the cavity mode (a) and I_b of the exciton mode (b) as functions of the laser detuning δ_a from the cavity (normalized to g), when the cavity–exciton detuning is $\delta_a - \delta_b = g$.

In figure 2, we show the variation of the intracavity electromagnetic field (a) and the excitonic field intensity (b) with the same values of the parameters as above and away from cavity–exciton resonance ($\delta_b - \delta_a = g$), as functions of $\delta = \delta_a$. The exciton-like peak of the cavity field corresponds to the left-hand-side peak of figure 2(a), whereas the cavity-like peak corresponds to the right-hand-side peak.

4. Quantum fluctuations and squeezing spectra

We deal with electromagnetic and exciton fields that have large average values compared with the fluctuations. We then write

$$\hat{a}(t) = a_0 + \delta\hat{a}(t) \tag{20}$$

$$\hat{b}(t) = b_0 + \delta\hat{b}(t) \tag{21}$$

where a_0 and b_0 are the classical mean values derived in the previous section and $\delta\hat{a}(t)$ and $\delta\hat{b}(t)$ are the quantum fluctuations of the field inside the cavity that we want to determine.

We use the Fourier transforms of the fluctuations defined as

$$\delta\hat{a}(\omega) = \int \delta\hat{a}(t)\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}t \tag{22}$$

$$\delta\hat{a}^{\dagger}(\omega) = \int \delta\hat{a}^{\dagger}(t)\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}t \tag{23}$$

where ω is a dimensionless frequency normalized to the inverse round-trip time in the cavity. The frequencies ω over which the fields and their fluctuations vary appreciably are of the order of the relaxation rates and of the exciton–photon coupling rate g, which are much smaller than the optical frequency ω_L. We linearize equations (3), (4) and we take their Fourier transform according to equations (22), (23). This allows us to replace the differential equations (3), (4) by linear algebraic equations.

The squeezing spectra, which can be easily measured in the outgoing light with a radio-frequency spectrum analyser connected to photodetectors, are directly related to the solutions of the linearized equations in the frequency domain $\delta\hat{a}^{\mathrm{out}}(\omega)$ and $\delta\hat{a}^{\mathrm{out}\dagger}(\omega)$. Indeed experiments allow us to measure the fluctuations of the output electric field in a quadrature defined by an angle θ with respect to some phase reference:

$$\delta\hat{x}_{\theta}^{\mathrm{out}}(\omega) = \mathrm{e}^{-\mathrm{i}\theta}\delta\hat{a}^{\mathrm{out}}(\omega) + \mathrm{e}^{\mathrm{i}\theta}\delta\hat{a}^{\mathrm{out}\dagger}(\omega) \tag{24}$$

and the measured spectra are given by

$$S_{\theta}(\omega) = \langle\delta\hat{x}_{\theta}^{\mathrm{out}}(\omega)\,\delta\hat{x}_{\theta}^{\mathrm{out}}(\omega)\rangle. \tag{25}$$

The details of the calculation have been published elsewhere [26]. We will concentrate here on the physical interpretation of the results. For any given cavity–exciton detuning, the spectra can be calculated either for a fixed laser detuning as a function of the fluctuation frequency ω, or at a fixed value of ω as a function of the laser detuning. As the second case is better suited to the experimental conditions, we will show the calculated spectra for $\omega = 0$ as a function of the laser detuning. Spectra will be shown in two cases: 'optimum' squeezing spectra, for which θ is adjusted at each point in such a way that it gives the minimal amount of noise and intensity squeezing spectra for which the quantum fluctuations are calculated for the amplitude quadrature of the output field.

Figure 3 shows a set of optimum squeezing spectra (left) and intensity squeezing spectra (right) at $\omega = 0$, as a function of the laser frequency δ, for cavity–exciton resonance ($\delta_a = \delta_b$), with γ_a, γ_b, g and I_m as in figure 2 and three different situations corresponding to increasing temperatures: (a) at zero temperature, $\langle n\rangle = 0$, (b) $\langle n\rangle = 0.2$, (c) $\langle n\rangle = 1$. It can be seen that a squeezing of about 30% is predicted in the absence of phonon scattering, even with the low values of the non-linearity that have been assumed. Squeezing is observable on the intensity, even though it is lower than the optimum squeezing. Excess noise observed on the intensity is the counterpart of squeezing: quantum fluctuations can only be reduced in some quadrature if they are increased in another quadrature. In the presence of small phonon interaction, squeezing can still be observed, but tends to disappear rapidly as $\langle n\rangle$ increases.

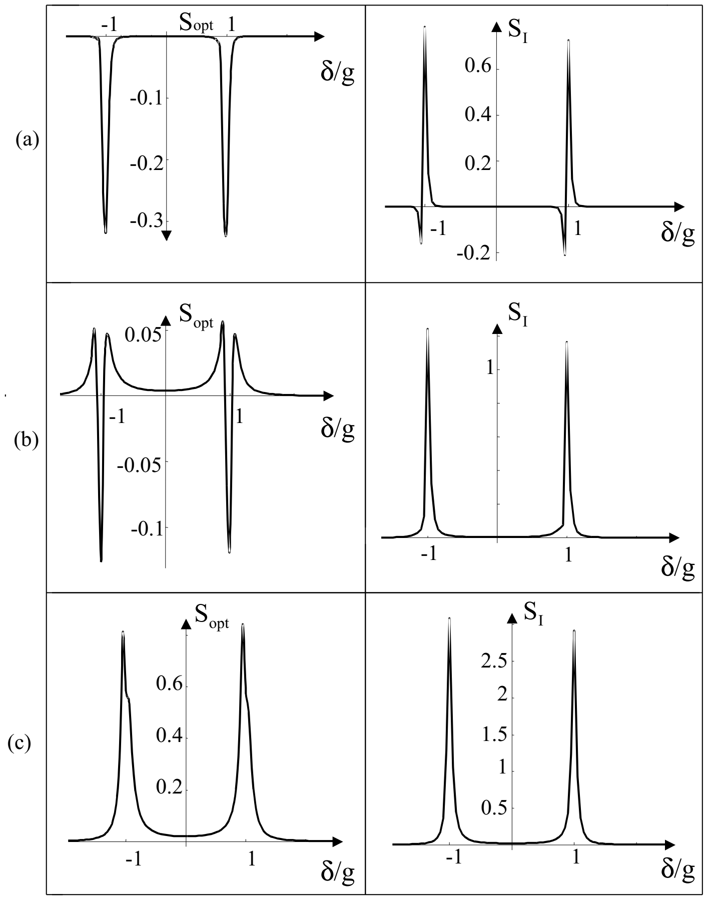

Figure 3. Optimum squeezing S_{opt} and intensity squeezing S_I (calculated at zero-noise frequency) as functions of the laser detuning δ (normalized to g), when the cavity is resonant with the exciton for the parameters of figure 1. The standard quantum noise corresponds to $S = 0$; perfect squeezing corresponds to $S = -1$. Curves (a), (b) and (c) correspond respectively to mean phonon numbers $\langle n \rangle = 0$, 0.2 and 1.

The coupling to a thermal bath brings additional fluctuations into the system that appear as excess noise on the output field. This excess noise is detrimental to the observation of quantum effects. However, it is expected that the study of this noise can provide interesting information on the relaxation processes in the semiconductor microcavity.

Figure 4 gives the same spectra as figure 3, but for a non-zero exciton–cavity detuning ($\delta_a - \delta_b = g$) as a function of $\delta = \delta_a$. It can first be seen that at zero temperature, squeezing appears mostly on the cavity peak. On the other hand, excess noise is largest on the exciton peak.

These curves have been calculated for excitonic densities of the order of 10^9 cm^{-2}, with one quantum well in the microcavity. With lower densities, the predicted squeezing scales down with the non-linear phase shift. On the other hand, if one wants to increase the squeezing, it may be difficult to increase the excitonic density because density-dependent relaxation effects neglected here will come into play. By increasing the number of quantum wells in the cavity, it is however possible to increase the non-linear phase shift while keeping the excitonic density

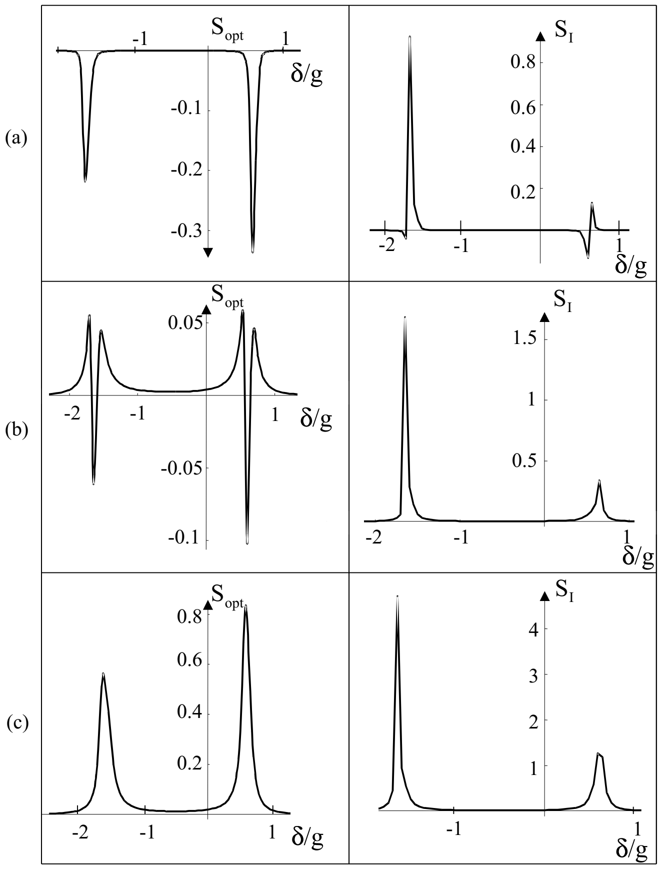

Figure 4. Optimum squeezing S_{opt} and intensity squeezing S_I (calculated at zero frequency) as functions of the laser detuning δ_a from the cavity (normalized to g), for the parameters of figure 2, when the cavity–exciton detuning $\delta_a - \delta_b = g$. The standard quantum noise corresponds to $S = 0$; perfect squeezing corresponds to $S = -1$. Curves (a), (b) and (c) correspond respectively to mean phonon numbers $\langle n \rangle = 0$, 0.2 and 1.

constant in each quantum well.

These results show that quantum effects should be observed in semiconductor microcavities at low temperature or in a system decoupled from the phonons. The decoupling of the lower polariton branch from relaxation has been predicted [16, 24] and observed in recent experiments [25] and indicates good prospects for such experiments.

The exploration of the noise should thus provide an interesting insight into the various effects that are involved in the build-up and the destruction of quantum features. In order to predict the expected phenomena accurately, a more elaborate model for the relaxation of the polariton is needed to deal with the case in which the two polariton branches are coupled in a different way to the phonons and to other excitons.

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